INTEGRATING USER RESPONSES TO PRICING IN DYNAMIC NETWORK MICRO-ASSIGNMENT MODELS

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Two Key Motivating Phenomena...

Growing Congestion....
Highway Authorities....
Why Road Pricing?

- **Motivating Phenomena:**
  - Growing congestion in metropolitan areas…
  - Budget constraints for highway authorities…

- **Objectives of road pricing:**
  - **Revenue generation:** road/bridge tolls
  - **Congestion management:** congestion pricing, cordon tolls and high occupancy toll (HOT) lanes…toward **dynamic pricing (with time-varying tolls)**…

- **Examples of road pricing applications**
  - **London cordon pricing:** charging private vehicles in downtown area to reduce traffic congestion and raise revenues for transport improvements.
  - **I-15 HOT lanes in San Diego:** allowing solo drivers to pay a dynamic toll to use the express lanes normally reserved for high occupancy vehicles (HOV).
  - **Highway 407, the Express Toll Route (ETR), in Toronto:** collecting tolls based on distance traveled in the multi-lane electronic highway.
  - **State Route 91 in Orange County, California:** express toll lanes constructed and operated by private company.
Rationale for Congestion Pricing

1. Market-clearing prices: charge whatever it takes to achieve desired service levels – use prices instead of wasted time/queues to rationalize use of transport infrastructure; efficiency argument.

2. To induce more efficient use of transport infrastructure—

   Network Equilibrium Theory: Use pricing to induce (Time-minimizing) system optimal (SO) flow pattern instead of inefficient user equilibrium (UE) attained without pricing.

   First-best pricing: Charge users marginal cost (imposed on system) on all network links;

   Second-best pricing: Impose tolls only on selected links (usually for practical reasons)
Congestion Pricing as Demand Management Tool

1. Pricing increasingly viewed as one instrument along with two main other controls for integrated transportation system management:
   1. Traffic controls: ramp metering, signal coordination
   2. Information Supply: advanced traveler information systems, parking information systems, variable message signs (VMS)…

2. In real-time: with improved sensing and information technologies, can determine prices, traffic controls and information strategies adaptively, online, based on current and anticipated state of the system
Implications for Evaluation Methodology

1. Consideration of time-variation (within day) of traffic demand and during peak-periods: dynamic analysis
2. Network perspective: cannot consider highway facility in isolation; need to consider traffic distribution across paths in a network
3. Need to capture congestion phenomena and queueing
4. Representation of operational aspects associated with coordinated measures: e.g. HOV lanes
5. User responses to prices:
   1. Short-term: route choice
   2. Medium-term: trip timing, mode choice
   3. Longer-term: destination choice, forsake trip (or telecommute); location and activity decisions
Network Simulation-Assignment Modeling for Advanced Traffic System Management
Why is DUE Model important for Dynamic Pricing Applications?

Estimated (dynamic) origin-destination (OD) demand

Dynamic User Equilibrium Models (or Dynamic Traffic Assignment)
- Address the dynamic nature of traffic flows
- Describe path choices of network users based on the user equilibrium/optimal
- Obtain DUE path flow patterns

Time-varying road pricing scenario (as a set of time-dependent link tolls)

Evaluation of dynamic pricing scenarios: predicted path choices (toll road usage, profit and revenue) and network performance (average travel time and average delay).

Decision support for toll operators and traffic system managers
User Heterogeneity

- Critical limitation of existing dynamic traffic assignment tools
  - Each trip-maker chooses a path that minimizes the two major path travel criteria: travel time and out-of-pocket cost (path generalized cost).
  - Conventional traffic assignment models consider a homogeneous perception of tolls by assuming a constant VOT in the path choice model.
  - Empirical studies (e.g. Hensher, 2001; Brownstone and Small 2005; Cirillo et al. 2006) found that the VOT varies significantly across individuals.

Path A: 45 minutes + $2
Path B: 55 minutes + $0
Our Objective

- Develop Multi-Criterion Simultaneous Route and Departure Time User Equilibrium (MSRDUE) models and algorithms
  - Address the heterogeneous user preference of path and/or departure time choices in response to time-varying toll charges.
  - Capture traffic flow dynamics and spatial and temporal vehicular interactions (simulation-based approach).
  - Adhere to the time-dependent generalization of Wardrop’s UE principle (gap function measures the deviation from equilibrium).
  - Be deployable on road traffic networks of practical sizes (vehicle-based implementation technique).
Problem Statement

Assumptions:

- $G(N, A)$, discretized planning horizon, and time-dependent link tolls.
- Define schedule delay as the difference between actual and preferred arrival times (PAT).
  - Every trip-maker has his/her own PAT interval $\theta$
  - Early schedule delay (ESD) and late schedule delay (LSD)
  - Value of ESD (VOESD $\beta$) and value of LSD (VOLSD $\lambda$)
- The experienced trip cost perceived by a trip-maker with $\theta$, $\alpha$, $\beta$, and $\lambda$

$$G_{odp}^\tau (\theta, \alpha, \beta, \lambda) = TC_{odp}^\tau + \alpha \times TT_{odp}^\tau + \beta \times ESD_{odp}^\tau (\theta) + \lambda \times LSD_{odp}^\tau (\theta)$$

Path generalized cost Schedule delay cost

where

$$ESD_{odp}^\tau (\theta) = \max \{0, \theta^{lb} - \tau^{mid}\} \quad LSD_{odp}^\tau (\theta) = \max \{0, \tau^{mid} - \theta^{ub}\}$$

- VOT $\alpha$, VOESD $\beta$, and VOLSD $\lambda$ are continuously distributed across trip-makers with given probability density functions and feasible ranges.
Problem Statement (ctd.)

- Departure time and path choice behavioral assumption:
  - Each trip-maker chooses the alternative that minimizes the experienced trip cost with respect to his/her PAT, VOT, VOESD, and VOLSD.
  - An alternative is a combination of arrival time interval and the corresponding least generalized cost path (that arrives the destination at that arrival time interval).

- Multi-criterion simultaneous route and departure time UE (MSRDUE)
  - For each OD pair, cannot decrease the experienced trip cost (given user’s particular VOT, VOESD, VOLSD, and PAT interval) by unilaterally changing departure time and/or path.
  - Each trip-maker is assigned to the alternative that has the least trip cost with respect to his/her own PAT, VOT, VOESD, and VOLSD.

- MSRDUE problem:
  Under a given time-dependent road pricing scenario, solve for the departure time and path flow patterns satisfying the MSRDUE conditions.
Why is this problem difficult?

- Relaxation of VOT from constant to continuous random variable
  - Find an equilibrium state resulting from the interactions of (possibly infinite) many classes of trips, each of which corresponds to a class-specific VOT.
  - Computing and storing such a grand path set is computationally intractable and memory intensive in (road) network applications of practical sizes
- Parametric Analysis Method (PAM) to find the set of extreme efficient (or non-dominated) path trees
  - In the disutility minimization-based path choice modeling framework with convex disutility functions
  - All trips would choose only among the set of extreme efficient paths
  - Applications in static assignment (Dial, 1996; Marcotte, 1997)
Column Generation-based MSRDUE algorithm

Input
OD demand, link tolls, VOT distribution, and initial path assignment

Initialization
Traffic simulation to evaluate initial path assignment

Sequential Parametric Analysis Method (SPAM)
VOT, VOESD, VOLSD breakpoints defining multi-user classes and extreme efficient alternatives for each class

Convergence Checking?
YES
Output path flows and terminate
NO

Multi-class Path Flow Updating Scheme

Multi-class Dynamic Network Loading
Traffic Simulation

Convergence Checking?
YES
NO

Outer Loop: Alternative Generation

Inner Loop: Equilibration
Determine VOT, VOESD, and VOLSD breakpoints that define multi-user classes, and find the least trip cost (extreme non-dominated) alternative for each user class.

Repeat the two stages for each destination: \( d = 1, \ldots, D \)

Stage 1: parametric analysis of VOT

Stage 2: parametric analysis of VOESD

Repeat the second stage for each VOT subinterval: \( b = 1, \ldots, 3 \)
Determine the breakpoints that partition the feasible VOT range and define the master user classes, and find time-dependent least generalized cost path tree for each user class.

- Each tree consists of time-dependent least generalized cost paths from all origin nodes to a destination node, for all arrival time intervals.
- To determine the subinterval of VOT, in which the current tree $Tr(\alpha)$ is optimal.
Given a time-dependent extreme efficient path tree $\mathcal{T}_r(b)$ corresponding to the VOT subinterval $[\alpha^{b-1}, \alpha^b)$, the parametric analyses of VOESD and VOLSD are conducted in an expanded network.
An example
Output of the SPAM

VOESD breakpoints that define the subintervals, and the least trip cost alternative for each subinterval. \( \forall b, \forall \theta, \)

\[
\beta(b, \theta) = \{\beta^0, \beta^1, ..., \beta^{M(b,\theta)} \mid \beta^{\text{max}} = \beta^0 > \beta^1 > ... > \beta^m > ... > \beta^{M(b,\theta)} = \beta^{\text{min}} \}\]

\[ [\beta^{m-1}, \beta^m)_{b,\theta}, (\tau^*, p^*)_{b,\theta,m}, \ m = 1, ..., M(b,\theta) \]

VOLSD breakpoints that define the subintervals, and the least trip cost alternative for each subinterval. \( \forall b, \forall \theta, \)

\[
\lambda(b, \theta) = \{\lambda^0, \lambda^1, ..., \lambda^{N(b,\theta)} \mid \lambda^{\text{max}} = \lambda^0 > \lambda^1 > ... > \lambda^n > ... > \lambda^{N(b,\theta)} = \lambda^{\text{min}} \}\]

\[ [\lambda^{n-1}, \lambda^n)_{b,\theta}, (\tau^*, p^*)_{b,\theta,n}, \ n = 1, ..., N(b,\theta) \]

Multiple user classes: for each VOT subinterval \( b \) and PAT \( \theta, \)

\[
u(b, \theta, m_{\beta(b,\theta)}, n_{\lambda(b,\theta)}), \ m = 1, ..., M(b,\theta), \ n = 1, ..., N(b,\theta) \]

Simplified as \( u(b, \theta, m, n) \)

The corresponding set of least trip cost alternatives

\[
alt_{od}(b, \theta, m, n) = alt_{od}(b, \theta, m_{b,\theta}) \cup alt_{od}(b, \theta, n_{b,\theta})
\]
Multi-Class Flow Updating and Convergence Checking

- **Multi-Class Alternative Flow Updating Scheme**
  - **Multiple user classes** $u(b, \theta, m, n)$ are naturally determined by the SPAM.
  - Decomposes the problem into many $(b, \theta, m, n, o, d)$ sub-problems and solves each of them by adjusting OD flows between non-least trip cost alternatives and the least trip cost alternative.
  - Extension of the multi-class path flow updating scheme for the BDUE

- **Convergence Checking**
  - **Gap**
    \[
    Gap(r^l) = \sum_{u(b,\theta,m,n)} \sum_{o} \sum_{d} \sum_{(\tau,p)\in alt_{od}(b,\theta,m,n)} r_{odp}^{\tau,l}(b,\theta,m,n) \times \Delta_{odp}^{\tau,l}(b,\theta,m,n)
    \]
  - **Average Gap**
    \[
    AGap(r) = \frac{\sum_{u(b,\theta,m,n)} \sum_{o} \sum_{d} \sum_{(\tau,p)\in alt_{od}(b,\theta,m,n)} r_{odp}^{\tau,l}(b,\theta,m,n) \times \Delta_{odp}^{\tau,l}(b,\theta,m,n)}{\sum_{u(b,\theta,m,n)} \sum_{o} \sum_{d} \sum_{(\tau,p)\in alt_{od}(b,\theta,m,n)} \sum_{(\tau,p)\in alt_{od}(b,\theta,m,n)} r_{odp}^{\tau,l}(b,\theta,m,n)}
    \]
Numerical Experiments and Results

- **Purpose**
  - Examine the algorithmic convergence property and solution quality of the algorithm.
  - Investigate how the random parameters would affect departure time and path flow patterns (or toll road usage) under different dynamic pricing scenarios (i.e. to compare the random and constant parameter models).

- **Random parameters**
  - VOT distribution: $N(0.4/\text{min}, 0.2/\text{min})$, $[\alpha^{\text{min}}, \alpha^{\text{max}}] = [0.01, 3.0]$ (Lam and Small, 2001; Brownstone and Small, 2005; Southern CA)
  - VOESD distribution: $N(0.3/\text{min}, 0.15/\text{min})$, $[\beta^{\text{min}}, \beta^{\text{max}}] = [0.01, 2.0]$  
  - VOLSD distribution: $N(1.8/\text{min}, 0.6/\text{min})$, $[\lambda^{\text{min}}, \lambda^{\text{max}}] = [0.25, 4.0]$ (economic judgments based on the results reported in Small (1982))

- **Arrival time and PAT intervals**: 5 minutes.
Experiment conducted on the Fort Worth network (TX)

- Select a critical OD pair that accounts for 25% of total demand.

**Pricing Scenario**

<table>
<thead>
<tr>
<th></th>
<th>0-20 minutes</th>
<th>20-40 minutes</th>
<th>40-60 minutes</th>
<th>60-80 minutes</th>
<th>80-100 minutes</th>
<th>100-120 minutes</th>
<th>120-150 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 (low)</td>
<td>$0.05</td>
<td>$0.20</td>
<td>$0.35</td>
<td>$0.50</td>
<td>$0.35</td>
<td>$0.20</td>
<td>$0.05</td>
</tr>
<tr>
<td>#2 (mid)</td>
<td>$0.25</td>
<td>$0.40</td>
<td>$0.55</td>
<td>$0.70</td>
<td>$0.55</td>
<td>$0.40</td>
<td>$0.25</td>
</tr>
<tr>
<td>#3 (high)</td>
<td>$0.45</td>
<td>$0.60</td>
<td>$0.75</td>
<td>$0.90</td>
<td>$0.75</td>
<td>$0.60</td>
<td>$0.45</td>
</tr>
</tbody>
</table>

**Dynamic pricing scenarios**
Numerical Experiments and Results

- Experiment conducted on the Fort Worth network (TX)
  - Convergence pattern and solution quality in terms of Average Gap.
  - Convergence pattern in terms of departure time distribution
Experiment conducted on Fort Worth network (TX)
- Convergence pattern in terms of the number of schedule delay vehicles (i.e. early, late, and on-time vehicles) in the random parameter model
Numerical Experiments and Results

- Experiment conducted on the Fort Worth network (TX)
  - Compare the differences in departure time distribution and toll road usage between random and constant parameter models

![Graph 1: Departure Time Distribution](image1.png)

![Graph 2: Time-Varying Toll Road Usage](image2.png)

departure time distribution

Time-varying toll road usage
Experiment conducted on the Fort Worth network (TX)

The Comparison of departure time distribution and toll road usage under different dynamic pricing scenarios

departure time distribution

Time-varying toll road usage
VOT and VOSD are assumed continuously distributed across trip-makers.

The MSRDUE problem is solved by the column generation-based algorithm which embeds the extreme non-dominated path finding algorithm – SPAM (sequential parametric analysis method), in addition to the multi-class alternative flow updating scheme and the traffic simulator.

The algorithm is independent of the VOT, VOESD, and VOLSD assumptions, and independent of the traffic simulator.

- The convergence pattern of the proposed MSRDUE algorithm is not affected by the different assumptions of VOT, VOESD, and VOLSD, and it is able to find close-to-MSRDUE solutions.
Concluding Remarks

- There are significant differences in the estimated/predicted departure time pattern and toll road usage between the two models.
  - Trip-makers behave identically in choosing departure times and paths in the constant parameter model.
  - The random parameter model explicitly considers heterogeneous users with different parameters.

- The proposed MSRDUE model can realistically describe trip-makers’ responses to time-varying toll charges in temporal distribution (departure times) and spatial splits (path flows).

- Future Works:
  - Inclusion of other path choice attributes, such as reliability.
  - Extensions to OD-specific and time-varying VOT, VOESD, and VOLSD distributions
  - Development of re-optimization algorithms for the PAM and SPAM
  - Applications to dynamic congestion pricing problems
Congestion Pricing as Demand Management Tool

1. Pricing increasingly viewed as one instrument along with two main other controls for integrated transportation system management:
   
   1. Traffic controls: ramp metering, signal coordination
   2. Information Supply: advanced traveler information systems, parking information systems, variable message signs (VMS)…

2. In real-time: with improved sensing and information technologies, can determine prices, traffic controls and information strategies adaptively, online, based on current and anticipated state of the system
Hot lanes will only be considered with the addition of a lane to the Beltway. No general purpose lanes will be converted to HOT lanes.

Single Occupant Vehicles allowed to use HOV lanes for a toll

Toll rates vary based on traffic conditions or time of day so as to maintain high level of service on managed lane

Facilitated by AVI and automatic toll collection
Value Pricing

- Value pricing
  - Let travelers choose between two adjacent roadways: priced but free-flowing vs. free but congested

- Applications
  - Predetermined toll values
    - SR-91 in Orange County, California
    - Harris County, Texas
  - Reactive
    - I-394 Minnesota
    - I-15 FasTrak in San Diego, California
Motivation

- Anticipatory pricing
  - Set toll values based on predictive traffic measures in order to prevent traffic breakdown before it occurs

- Managed lanes
  - Anticipatory ramp metering

- Advanced Traveler Information Systems (ATIS)
  - Anticipatory travel time information provision

- Anticipatory measures are expected to be more effective than the prevailing measures when prediction is reliable
Reactive Pricing Strategy

- How does reactive pricing work?
  - obtain the prevailing traffic measures/conditions
  - adjust current link tolls accordingly
  - communicate to drivers via local VMS at the entry point
  - could also disseminate via radio, in-vehicle equipment, mobile, internet etc.
What differentiates anticipatory from reactive pricing?
- Network state prediction
- Use predicted traffic conditions
- Calculate link toll within the prediction horizon and implement it in real time
Toll generation problem is formulated as a mathematical programming with equilibrium constraints (MPEC)

Embedded in a closed-loop rolling horizon framework

MPEC problem is solved within the prediction horizon
Rolling horizon approach is a practical method for real-time demand-responsive control.

Basic idea: Vehicles currently assigned will not be influenced by vehicles assigned far into the future.
Objective function
- Minimize the total travel time

Subject to the constraints
- Maintain high LOS on the toll links (free flow speed)
- Second best toll: on selected links only
- Feasible link flow
- Flow conservation
- Tolled user equilibrium conditions
Methodology

- Step 0: start from time 0 and set initial link tolls
- Step 1: UE traffic assignment with given current link tolls
- Step 2: update link toll range based on prevailing/predicted concentration
- Step 3: search for second-best toll values (within the toll range) by gradient based algorithm
- Step 4: if the end of planning horizon is not reached, roll forward and go to Step 1; otherwise, stop
Tolled UE Traffic Assignment

- Given a set of link tolls
- Search for tolled UE flow by
  - an iterative procedure
  - or, simulating users’ route switching decisions based on real-time information and bounded rationality
    - Time-dependent travel time
    - Time-dependent link toll
    - Generalized cost = travel time + link toll
    - Switch path if the improvement in the remaining trip cost exceeds the indifference band
The minimum and maximum toll values

\[ \beta_{\text{min}}^{la} = \beta^{(t-1)a} + \alpha \cdot \sum_{i=0}^{r} w_i (c^{(t+i)a} - c_{cr}^a) \]
\[ \beta_{\text{max}}^{la} = \beta^{(t-1)a} + \alpha \cdot \sum_{i=0}^{h-1} w_i (c^{(t+i)a} - c_{bp}^a) \]

Within feasible range
- Non-negative
- Preset upper bound for link toll
Second-best Toll Search

- Simplify the problem as a single-level optimization problem

\[
\min_{\beta} \theta(\beta, x(\beta))
\]

subject to \( \beta \in \Omega \)

- Projected sub-gradient approach (PSA) to search for good local optima
  - Evaluate gradient at \( \beta_k \)
  - Define search direction \( s^k \)
  - Decide step length \( l^* \)
  - Update toll value

\[
\beta_{k+1} = \text{Pr}_\Omega (\beta_k + l^* s^k)
\]
For the **reactive** case: activate the state estimation module, which interacts with the reactive toll generator.

For the **anticipatory** case: activate both estimation and prediction modules, which work together with the anticipatory toll generator.
The Test Bed Network: CHART

- I-95 corridor between Washington, DC and Baltimore, MD, US
- 2 toll lanes
- 2241 nodes
- 3459 links
- 111 TAZ zones
- 2 hours morning peak demand
Pricing Strategies

- No pricing (base case)
- Static pricing
  - Predetermined the time-varying link tolls based on the historical information
- Reactive pricing
  - Set time-varying link tolls based on prevailing traffic conditions
- Anticipatory pricing
  - Set time-dependent link tolls based on predicted traffic conditions
Preliminary Results – Travel Time

- Warm-up period: increase in travel time at the beginning
- With the anticipatory pricing strategy, the travel times become steady after 1 hour (free flow condition)
- Static pricing strategy provides free flow condition on the toll lanes, but reduces the LOS on the alternative freeway lanes
Concentrations averaged over links along the congested portion of toll road, weighted by the link length.

Throughputs measured at downstream of where traffic breaks down in base case (no pricing).

Anticipatory pricing strategy can provide higher throughput while maintaining lower concentration (steady traffic flow).
Conclusion

- The second-best dynamic congestion pricing problem was formulated as a MPEC in a closed-loop rolling horizon framework.
- Anticipatory pricing sets the toll values based on future traffic conditions so that it can prevent traffic breakdown in advance and maintain steady traffic flow.
- Many challenges remain—methodology, application, social acceptability.