Short-Term Freeway Traffic Flow Prediction: Bayesian Combined Neural Network Approach

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Abstract: Short-term traffic flow prediction has long been regarded as a critical concern for intelligent transportation systems. On the basis of many existing prediction models, each having good performance only in a particular period, an improved approach is to combine these single predictors together for prediction in a span of periods. In this paper, a neural network model is introduced that combines the prediction from single neural network predictors according to an adaptive and heuristic credit assignment algorithm based on the theory of conditional probability and Bayes' rule. Two single predictors, i.e., the back propagation and the radial basis function neural networks are designed and combined linearly into a Bayesian combined neural network model. The credit value for each predictor in the combined model is calculated according to the proposed credit assignment algorithm and largely depends on the accumulative prediction performance of these predictors during the previous prediction intervals. For experimental test, two data sets comprising traffic flow rates in 15-min time intervals have been collected from Singapore's Ayer Rajah Expressway. One data set is used to train the two single neural networks and the other to test and compare the performances between the combined and singular models. Three indices, i.e., the mean absolute percentage error, the variance of absolute percentage error, and the probability of percentage error, are employed to compare the forecasting performance. It is found that most of the time, the combined model outperforms the singular predictors. More importantly, for a given time period, it is the role of this newly proposed model to track the predictors' performance online, so as to always select and combine the best-performing predictors for prediction.

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Introduction

Short-term traffic flow prediction has long been regarded as a critical concern for intelligent transportation systems. In particular, such traffic flow forecasting supports (1) the development of proactive traffic control strategies in advanced traffic management systems (ATMSs); (2) real-time route guidance in advanced traveler information systems (ATISs); and (3) evaluation of these dynamic traffic control and guidance strategies as well. In an early report on the architecture of intelligent transportation systems (Cheslow et al. 1992), it was clearly indicated that the ability to make continuous predictions of traffic flows and link travel times for several minutes into the future, using real-time traffic data, is a major requirement for providing dynamic traffic control and guidance. For operational analyses, the *Highway Capacity Manual* (TRB 2000) suggests using a 15-min traffic flow rate. In this research, the flow rate of a 15-min time interval is hence taken as the study object.

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The short-term forecasting of traffic conditions has had an active but somewhat unsatisfying research history (Davis and Nihan 1991). Up to now a variety of methodologies have been applied to short-term traffic flow forecasting, including the multivariate time-series model (Ahmed and Cook 1979; Hamed et al. 1995; Williams et al. 1998), the Kalman filtering method (Okutani and Stephanedes 1984), the nonparametric regression model (Davis and Nihan 1991; Smith et al. 2002), and the neural network model (Smith and Demetsky 1994; Zhang et al. 1997; Dougherty and Kirby 1998; Park et al. 1998). Generally these techniques can be classified into statistical models (including regression models and time-series models) and artificial intelligence or neural network models. Comparison between these models (Kirby et al. 1997; Smith and Demetsky 1997; Smith et al. 2002), however, showed that no single predictor had yet been developed that presented itself to be universally accepted as the best, and at all times, an effective traffic flow forecasting model for real-time traffic operation.

Recently, a type of hybrid neural network model drew much attention, and brought encouraging results. Some examples like those of Park and Rilett (1998); Yin et al. (2002); Kuchipudi and Chien (2003) combined some clustering techniques with either time-series analysis or neural network models for integrated prediction. In these research activities, a neural network model was adopted to classify traffic flow pattern before an appropriate model was applied for modeling and prediction for each pattern. It was found that such a modular neural network model outperformed the singular neural network model as well as conventional models such as exponential smoothing and the

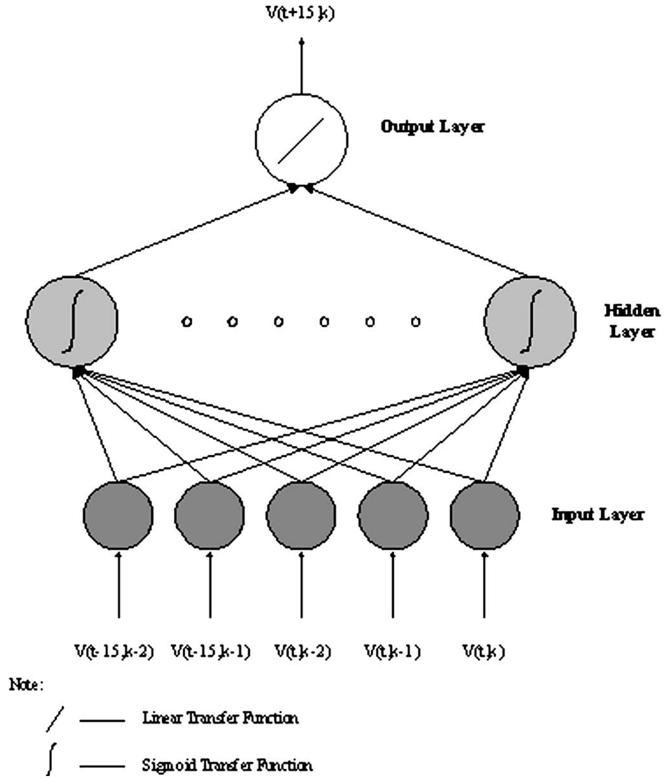

Fig. 1. Architecture of back propagation neural network

Kalman filtering method. As traffic flow itself is a complicated process influenced by many factors, it is found that using such hybrid models to describe and predict the effect of different factors on the traffic flow is appropriate.

It is an important observation to find that a predictor has superior performance for a particular time period, which means that the corresponding model yields a good representation of the underlying process that generates the short-term time series during that specific period. The existence of several models, each yielding a superior prediction performance for a particular period, makes it plausible that several different sources participate in the generation of the overall time series. On the basis of many predictors, each predictor having good performance only in a particular period, an improved approach is to combine the single predictors together for prediction in a span of periods. It is supposed that we will get improved performance by combining several predictors together also as a kind of modular or hybrid model and therefore arrive at more accurate prediction results in the whole period. In this paper, an adaptive forecasting method named the Bayesian combined neural network (BCNN) model is introduced, which combines several single neural network predictors and assigns a different credit value to each predictor, according to a proposed credit assignment algorithm that the model getting the highest credit acts as the main predictor for prediction in a given period. The advantages of these several models are integrated and the accuracy of prediction is thus improved.

The primary objective of this study is to develop and test a BCNN model for short-term freeway traffic flow prediction. Two single neural network predictors are selected and designed for single point traffic flow prediction, which is used to forecast the traffic volume for a site in the next time interval based on the observed traffic volumes in previous intervals, as well as from upstream locations. For this purpose, two data sets comprising traffic volume records in 15-min time intervals are hence collected from Ayer Rajah Expressway in Singapore. After being trained by one of the data sets, the two neural network predictors are combined into the BCNN model for prediction. Its prediction performance is tested and compared with those of single neural network predictors in terms of selected indices from another data set.

This paper contains five sections. Following the introduction, the second section provides the method of investigation of the Bayesian combination approach and the proposed BCNN model. The third section describes how the model thus derived is applied to short-term traffic flow prediction on the Ayer Rajah Expressway as a numerical example. In the fourth section, results from prediction experiments are presented and compared among all of these predictors for analysis. The last and fifth section presents concluding remarks on the findings of this research.

Method of Investigation

Bayesian Combination Approach

The Bayesian combination approach is a type of method that tries to combine several predictors based on the conditional probability and Bayes' rule (Petridis et al. 2001). Suppose that a traffic flow time series y_t is produced by one of the specific k time-series models y_t^k $(k=1,2,\ldots,K)$

$$y_t = y_t^k(y_{t-1},y_{t-2},\ldots,y_1) + e_t^k \tag{1}$$

where y_t=actual traffic flow rate in time interval t; y_t^k=kth forecasting model; e_t^k=corresponding forecast error. However, in each time interval, Eq. (1) will hold true for only one value of k and most of the time, the correct or "best" model cannot be identified in advance. A variable Z is therefore introduced to express this uncertainty, and Z is assumed to take one of the k values $(1,2,\ldots,K)$ in each time interval. Thus, the conditional posterior probability p_t^k $(k=1,2,\ldots,K;\ t=1,2,\ldots)$ is defined as

$$p_t^k = \mathrm{Prob}(Z=k/y_t,y_{t-1},\ldots,y_1) \tag{2}$$

With Bayes's rule (Montgomery and Runger 1999)

$$p_t^k = \frac{\mathrm{Prob}(y_t,Z=k/y_{t-1},\ldots,y_1)}{\sum_{m=1}^{K}\mathrm{Prob}(y_t,Z=m/y_{t-1},\ldots,y_1)} \tag{3}$$

and the fact that

$$\begin{aligned}&\mathrm{Prob}(y_t,Z=k/y_{t-1},y_{t-2},\ldots,y_1)\\&\quad=\mathrm{Prob}(y_t/y_{t-1},y_{t-2},\ldots,y_1,Z=k)\cdot p_{t-1}^k\end{aligned} \tag{4}$$

and assuming that $e_t^k=y_t-y_t^k$ is a Gaussian white noise time series with zero mean and standard deviation σ_k, then

$$\begin{aligned}\mathrm{Prob}(y_t/y_{t-1},\ldots,y_1,Z=k) &= \mathrm{Prob}(e_t^k=y_t-y_t^k/y_{t-1},\ldots,y_1,Z=k)\\ &= \frac{1}{\sqrt{2\pi}\sigma_k}e^{-[(y_t-y_t^k)/\sigma_k]^2}\end{aligned} \tag{5}$$

Combining Eqs. (3), (4), and (5) yields

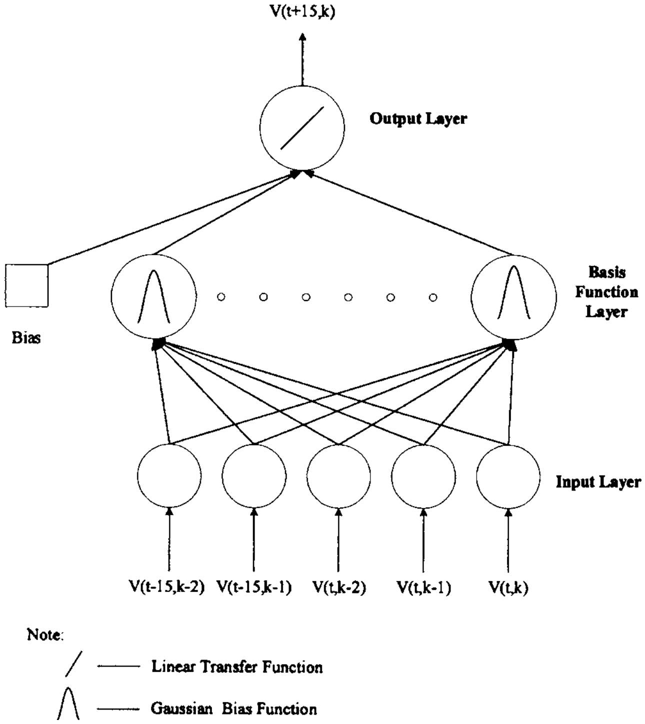

Fig. 2. Architecture of radial basis function neural network

$$p_t^k = \frac{\frac{1}{\sqrt{2\pi}\sigma_k} p_{t-1}^k \cdot e^{-[(y_t - y_t^k)/\sigma_k]^2}}{\sum_{m=1}^{K} \frac{1}{\sqrt{2\pi}\sigma_m} p_{t-1}^m \cdot e^{-[(y_t - y_t^m)/\sigma_m]^2}} \quad (6)$$

Eq. (6) expresses the probability that model k generates the observed traffic flow rate series, which is also the credit value assigned to the kth predictor in the combined model. Such a credit assignment algorithm is an adaptive and heuristic scheme, which depends on observations up to time t and the prediction performance of all predictors in previous intervals. On the other hand, it can be seen from Eq. (6) that the model which has a larger forecasting error $(y_t - y_t^k)^2$ is heavily penalized, resulting in a decreased p_t^k, which on the contrary means that the model that best forecasts the observed data currently will gain the highest p_t^k in the combined model and become the main predictor in the next time interval. Hence, the prediction result in time interval $t+1$ generated by the combined model is written as the linear combination of output of the K predictors as the following formula:

$$y_{t+1}^* = \sum_{k=1}^{K} P_t^k \cdot y_{t+1}^k \quad (7)$$

where y_{t+1}^*=prediction result in time interval $t+1$ given by the combined model; p_t^k=credit value of kth predictor from Eq. (6); y_{t+1}^k=prediction result in time interval $t+1$ from kth predictor.

Generally, any kind of model, such as the linear regression model, exponential smoothing model, or other time-series models, may be combined into this model, based on their forecasting performance, to form a modular model according to the above combination approach. As neural networks are modeling techniques that have the potential of accurately describing the behavior of nonlinear complex systems, such as a traffic flow system, the single predictor type to be combined in this paper is selected as the neural network, which builds a model called the BCNN model.

Bayesian Combined Neural Network Model Development and Application

In this research, the main task of prediction is defined as forecasting the traffic flow rate in the next time interval for a downstream location, based on the observed traffic flow rates in the former intervals as well as from the upstream locations. Two single predictors, the back propagation (BP) neural network and the radial basis function (RBF) neural network, are selected and designated for single point short-term traffic flow prediction initially. These two designed neural network predictors later are to be combined into the BCNN model for prediction.

Back Propagation Neural Network Model

Following a comparison among a group of models, a BP neural network is selected with the following architecture: five neurons in the input layer, one hidden layer with 20 neurons, and one output neuron. The input variables include the flow rate $V(t-15\ \mathrm{min}, k-2)$, $V(t-15\ \mathrm{min}, k-1)$, $V(t\ \mathrm{min}, k-2)$,

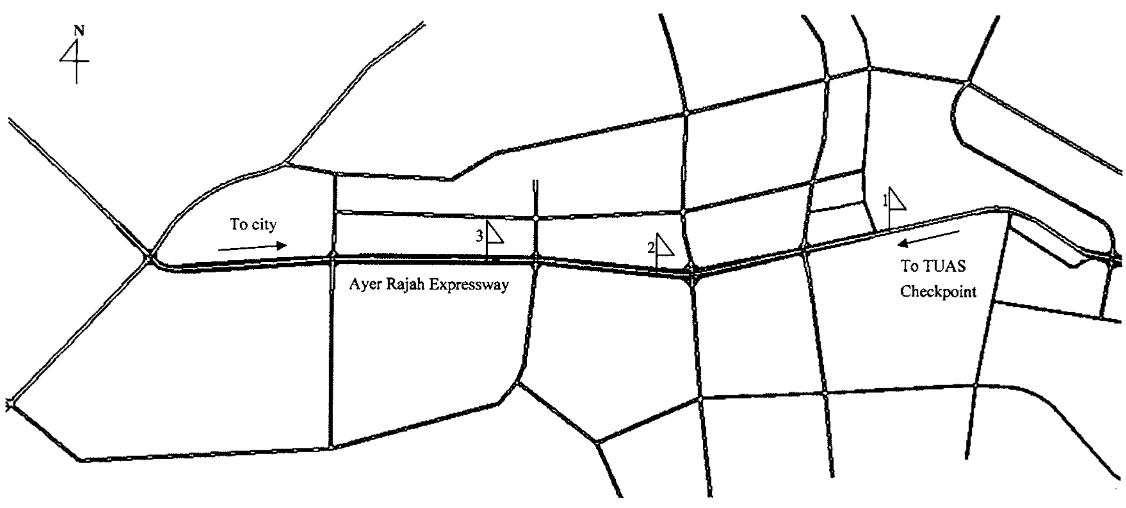

Fig. 3. Test road network: Ayer Rajah Expressway corridor in Singapore

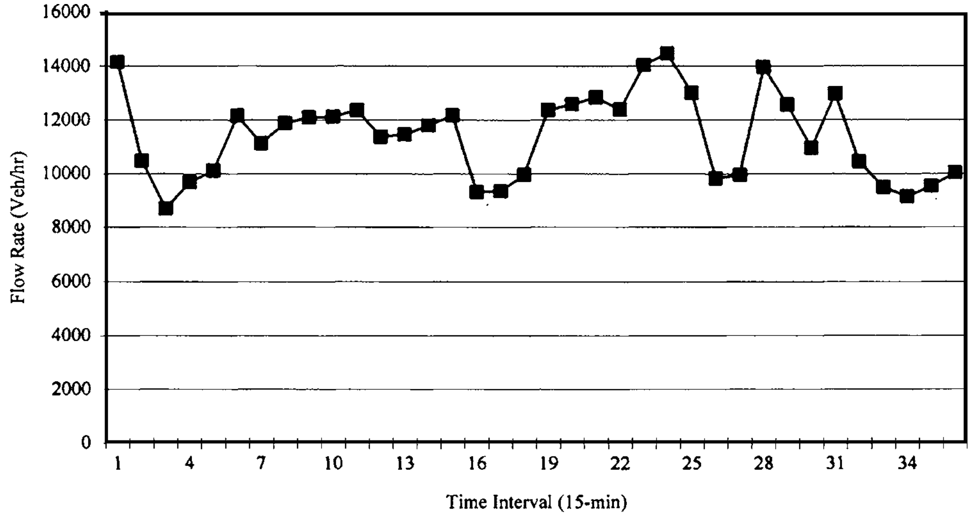

Fig. 4. Typical daily traffic flow pattern of Location 3

$V(t\ \min, k-1)$, and $V(t\ \min, k)$, while the output variable is $V(t+15\ \min, k)$, with t representing the current time and k representing the link section in the network; while $k-1$ denotes the nearer upstream link section and $k-2$ the further one. The model architecture is displayed as Fig. 1. The activation or transfer function in the hidden layer is chosen as the hyperbolic sigmoid function tanh.

Radial Basis Function Neural Network Model

The RBF neural network is in fact a kind of feedforward neural network with a single hidden layer, which uses a basis function to cluster the input vector and produce an output for further transfer with that input vector. In this study, the Gaussian function is chosen as the basis function with 45 cluster centers after the performance comparison of different architectures. The input and output variables are same as those of BP neural network. The developed architecture of this neural network is displayed as Fig. 2.

Bayesian Combined Neural Network Model

Based on the Bayesian combination approach theory, the developed two single neural network predictors are combined linearly into the BCNN model with a credit for each predictor. According to Eq. (6), the credit value is calculated as the posterior probability for the observed traffic flow time series based on the prediction performance of these two predictors as follows:

$$p_t^k = \frac{\frac{1}{\sqrt{2\pi\sigma}} p_{t-1}^k \cdot e^{-[(y_t - y_t^k)/\sigma_k]^2}}{\frac{1}{\sqrt{2\pi\sigma}} p_{t-1}^1 \cdot e^{-[(y_t - y_t^1)/\sigma_1]^2} + \frac{1}{\sqrt{2\pi\sigma}} p_{t-1}^2 \cdot e^{-[(y_t - y_t^2)/\sigma_2]^2}}$$

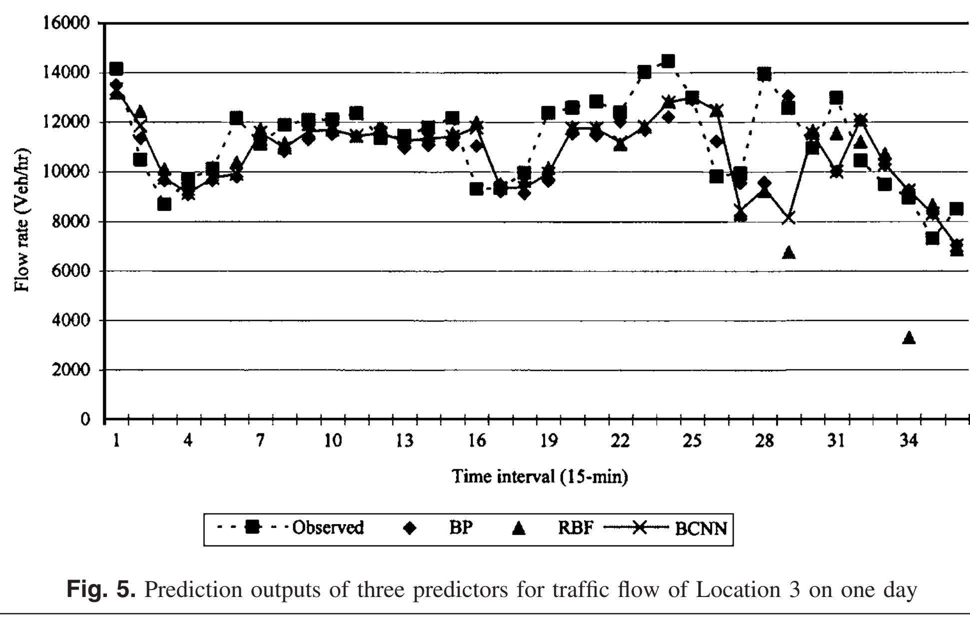

Fig. 5. Prediction outputs of three predictors for traffic flow of Location 3 on one day

$$k = 1,2 \tag{8}$$

Based on Eq. (8), the credit values for BP and RBF neural network predictors after a time interval t ($t=1,2,\ldots$), that is, p_t^1 and p_t^2, will be calculated iteratively, while p_0^1 and p_0^2 are chosen to be $1/2$ for simplification. The output of the BCNN predictor in time interval $t+1$ (y_{t+1}^*) is thus formulated as

$$y_{t+1}^* = P_t^1 \cdot y_{t+1}^1 + P_t^2 \cdot y_{t+1}^2 \tag{9}$$

where y_{t+1}^1 and y_{t+1}^2=respective prediction outputs of BP and RBF neural network predictors in time interval $t+1$.

Short-Term Traffic Flow Prediction on Ayer Rajah Expressway

The BCNN model built was applied to short-term traffic flow prediction on the Ayer Rajah Expressway in Singapore as a numerical example. In this experiment, two data sets, that is, the training set and the test set, were collected from three locations along the Ayer Rajah Expressway (Fig. 3). In these data sets, traffic flow rates in 15-min time intervals between 8:00 and 20:00 hours were collected for the three locations. Each record in the data set includes $V(t-15\text{ min},1)$, $V(t-15\text{ min},2)$, $V(t\text{ min},1)$, $V(t\text{ min},2)$, $V(t\text{ min},3)$ and $V(t+15\text{ min},3)$, in which t represents the current time, and the numbers (1,2,3) the observed locations.

The main task of prediction in this numerical experiment is defined as forecasting the traffic flow rate in the next time interval for the downstream location (Location 3), that is, $V(t+15\text{ min},3)$, based on the observed traffic volume data in the previous intervals as well as from upstream locations (Locations 1 and 2). A typical traffic flow pattern on a day on the downstream site is presented in Fig. 4. For the training set, traffic flow data from 16 days comprising a total of 692 records were prepared. These data were used to train the two single neural network predictors which later formed the BCNN prediction. Next, the data from four other observation days, comprising 152 records, were used to test BCNN performance and compare the performances of the following three models, i.e., the BP neural network, the RBF neural network, and the BCNN, after they were applied to the prediction for the test data set.

For the model training, the momentum learning rule with a learning rate of 0.1 was selected for both the BP and RBF neural network models. The stop criterion for the BP neural network model learning was that the mean square error for the cross validation data reached a threshold of 0.01, while the maximum epoch for the supervised learning was limited to 1000. For the RBF neural network model, learning includes two parts: unsupervised learning and supervised learning. The stop criterion for unsupervised learning was that the weight change reached a threshold of 0.0001 with a maximum epoch of 100. The stop criterion for supervised learning was same as that in the BP neural network model.

After the BP and RBF neural network models were trained, they were applied to the test data set for prediction. The prediction outputs were compared with the observed traffic flow data to test the performance of the two neural network models. The BCNN prediction for the traffic flow rate in the next time interval was based on the output value of the two neural network predictors in that interval, as well as the observed output value up to the current interval. In each time interval, the observed output was compared to the predicted outputs of the neural network predictors to determine the conditional posterior probability [Eq. (8)]. This probability represented the likelihood that one of the two models would generate the observed traffic flow rate series; and also acted as the credit value of each predictor in the BCNN for combined prediction in the next time interval [Eq. (9)]. The BCNN predicted output was then tested with the observed output. Finally, its performance was compared with those of the two neural network predictors.

Results and Analysis

Fig. 5 presents the prediction outputs of three predictors for the traffic flow rate of Location 3 on a typical day. The observed traffic flow on that day is also presented for comparison. As shown in the results, with the exception of the RBF model in the last few intervals, all three predictors showed a good reflection of the changing trends of traffic flow, while the BCNN predictor gave the best approximation of the actual traffic flow pattern.

Three indices, that is, the mean absolute percentage error (MAPE), the variance of absolute percentage error (VAPE), and probability of percentage error (PPE), were selected and employed to compare the forecasting performances of the three aforementioned models. As the MAPE and VAPE reflect the accuracy and stability of the predictor, the probability of percentage error, i.e., PPE, indicates the reliability of the prediction. The MAPE and VAPE are defined as follows:

$$MAPE = \frac{\sum_{t=0}^{N-1} \left(\frac{\text{abs}[V(t+1) - \hat{V}(t+1)]}{V(t+1)} \right)}{N} \tag{10}$$

$$VAPE = \sqrt{\frac{N\sum_{t=0}^{N-1} \left(\frac{\text{abs}[V(t+1) - \hat{V}(t+1)]}{V(t+1)} \right)^2 - \left[\sum_{t=0}^{N-1} \left(\frac{\text{abs}[V(t+1) - \hat{V}(t+1)]}{V(t+1)} \right) \right]^2}{N(N-1)}} \tag{11}$$

Table 1. Performance Comparison of MAPE, VAPE, and Probability of Percentage Error for Three Predictors

	BP neural network			RBF neural network			BCNN		
Time	MAPE (%)	VAPE (%)	Probability (error < ±10%) (%)	MAPE (%)	VAPE (%)	Probability (error < ±10%) (%)	MAPE (%)	VAPE (%)	Probability (error < ±10%) (%)
Day 1	6.90	6.83	77.8	5.41	4.40	91.7	5.81	4.94	94.4
Day 2	6.99	7.03	83.3	6.99	6.24	77.8	6.34	6.37	80.6
Day 3	7.13	8.50	75.0	6.18	6.76	86.1	6.07	6.86	83.3
Day 4	7.27	6.77	75.0	6.06	5.99	86.1	6.20	5.41	88.9
Total	7.08	7.24	77.8	6.16	5.86	85.0	6.10	5.81	86.9

where $V(t+1)$ = observed traffic volume in time interval $t+1$; $\hat{V}(t+1)$ = predicted traffic volume in time interval $t+1$; N = number of intervals for prediction.

Eq. (10) calculates the average relative error between the prediction output and actual observed data, which represents the accuracy of the prediction. The calculation of Eq. (11) represents the sum of the deviations from the average performance during the prediction in all intervals. It is obvious that a predictor with a large VAPE is not as stable as one with a smaller VAPE.

The traffic volumes collected from the four-day observation period were incorporated into the test data set and used for prediction and comparison among the three models which were built. The MAPE, VAPE, and probability of percentage error of these predictors are found in Table 1.

From Table 1, it can be seen that in the four-day prediction, the BCNN predictor has a better prediction performance than the other two single neural network predictors on most days in terms of accuracy and stability, which is indicated by their MAPE and VAPE values. Even if the performance of the BCNN predictor is not as good as those of the two predictors on some days, there is only a slight difference. This is mainly because on those days, one of the two predictors, that is, the RBF neural network model, yielded a better performance than the other, causing the combined model to be inclined to keep following the behavior of that model only. If each of the two predictors had a better performance during partial periods of a particular day, then the combined model would integrate their good performances together into a model with higher accuracy and better performance. It was also found that on nearly all four days, the BCNN gave a more reliable prediction, as it showed a probability of more than 85% (up to 90%) of yielding prediction outputs with a forecasting error margin of less than 10%. This was higher than those of the other two predictors. With such a level of accuracy, the combined model could be considered as suitable input for short-term traffic scenario construction for the whole network, to be used as the foundation and traffic environment for development of proactive traffic control strategies in ATMSs and real-time route guidance in ATISs. As a whole, the BCNN predictor performs better than the BP and RBF neural network predictors and is a potential model for field implementation.

Fig. 6 shows the distribution of the prediction error for the two single predictors, that is, the BP neural network and the RBF neural network, on one day. It can be seen that the performances of these two predictors are intertwined with each other. The BP neural network had a lower prediction error in the initial intervals and between time intervals 25 and 36. The RBF neural network gave a better performance at other time stages, that is, time intervals 4–25. However, in the last few time intervals, it had high errors. In practice, it is never known in advance which predictor would give the better prediction output in a specific period, so that that predictor could be chosen for prediction in the next time interval. Based on the prediction performance in the previous stages, the BCNN model selects and combines a better predictor for prediction in each interval, thus avoiding the erratic performance that may be caused by any single model. In this study, the

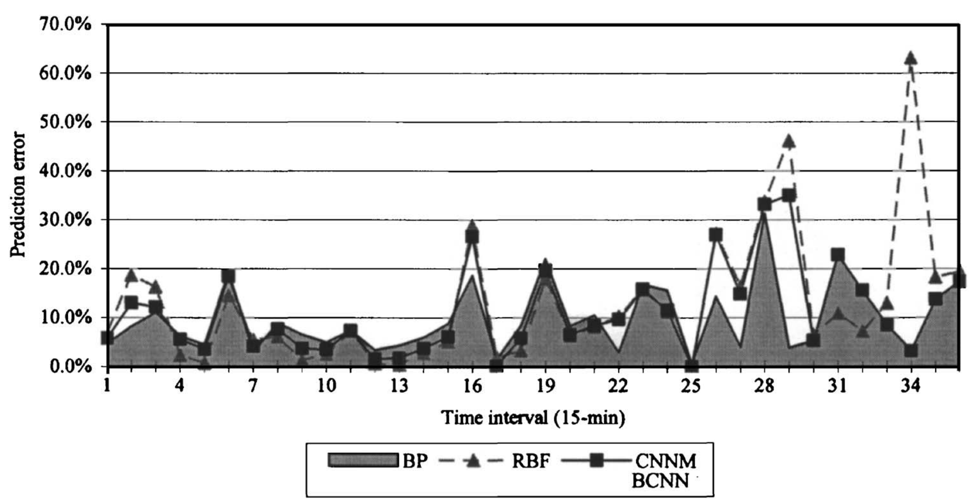

Fig. 6. Distribution of prediction errors for three predictors on one day

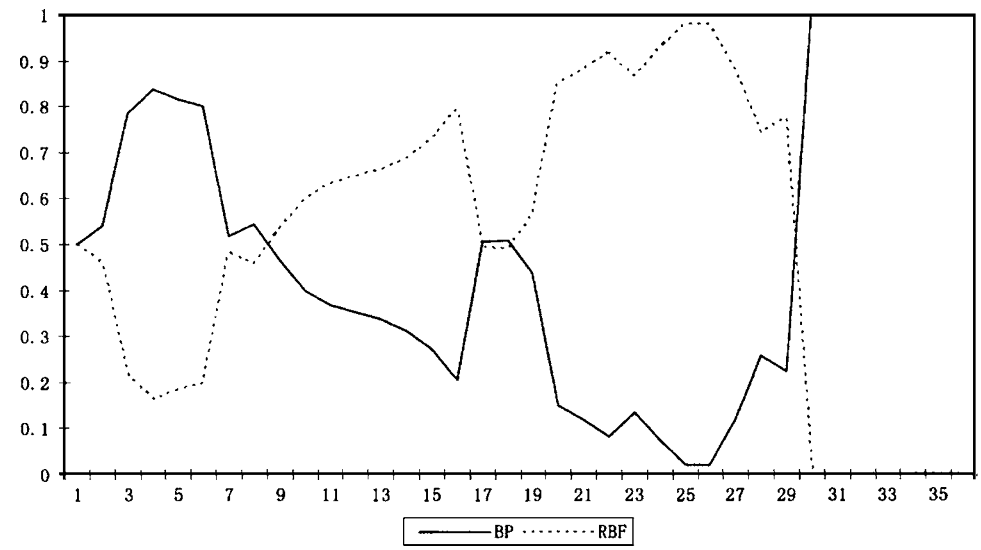

Fig. 7. Evolution of credit values of two predictors over prediction period

BCNN model followed the behavior of the RBF neural network predictor and kept low prediction error at the early stages. After detecting that the BP neural network predictor gave a better performance in the later intervals, the combined model switched to the BP neural network predictor, avoiding the high prediction error from the RBF neural network predictor. Therefore, it can be noted that the BCNN always maintains a low prediction error and results in a prediction output of high accuracy. Such a relationship can also be seen from the picture in Fig. 7, which shows the evolution of the posterior probabilities, that is, the credit values of the two predictors over the whole prediction period. As the BCNN relies on the accumulated performances in previous time stages and the observed outputs in the current time interval, it is an adaptive predictor.

Concluding Remarks

In this study, two neural network predictors and a combined neural network model known as the BCNN, which is based on the Bayesian combination approach, were developed for short-term freeway traffic flow prediction. It was found that for more than 85% time intervals, the proposed BCNN model outperformed the single predictors. Its mean absolute percentage error and variance of absolute percentage error were comparatively low. A fairly high probability for the prediction output to maintain an error margin of less than 10% was achieved by this combined predictor, which offers a good consideration for real time traffic operation and management. With the prediction outputs from this model as potential inputs, short-term traffic scenarios shall be constructed, which supplies a good support for ATMIS and ATIS operation. As it cannot be known in advance which particular predictor will yield the best prediction in a specific time interval, it is precisely the role of the BCNN model in tracking predictor performance online, and selecting and combining the best-performing predictors for prediction.

A further insight gained from this research is that the performance of the BCNN model heavily depends on the performance of the predictors it combines. Therefore, when more high-accuracy predictors appearing within a particular period are developed and combined, more accurate and stable prediction outputs will be generated by the BCNN predictor.

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